THE FREQUENCY-PULSE METHOD FOR DETERMINING THE THERMOPHYSICAL CHARACTERISTICS OF SOLID MATERIALS

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On the basis of a linear instantaneous heat source, we consider the frequency-pulse method of determining the thermophysical characteristics of solid materials without destruction of their integrity. We present the results for the investigation of the procedural error in the determination of thermophysical coefficients by the method of numerical simulation, as well as the results of experimental investigations.

In solving problems connected with operational control of the thermophysical characteristics of solids, of particular interest are nondestructive methods using a linear instantaneous heat source located on the surface of a thermally semi-infinite material [1].

The magnitude of excess temperature at an arbitrary point of the plane subjected to the influence of a heat pulse of infinitely small duration at the initial instant of time is described by the expression [2]

$$T(x, \tau) = \frac{Q}{2\pi\tau\lambda} \exp\left(-\frac{x^2}{4a\tau}\right), \quad \tau > 0, \tag{1}$$

where Q is the amount of heat liberated instantly per unit length of the source; x is the distance reckoned along the normal from the line of source action; τ is the time reckoned from the instant of heat pulse action.

The magnitude of excess temperature on the line of the disposition of the source ($x = 0$) is described by the expression

$$T(0, \tau) = \frac{Q}{2\pi\tau\lambda}, \quad \tau > 0. \tag{2}$$

When the heat source acts by heat pulses at equal time intervals τ', we may use Eq. (1) to obtain the following expression for the temperature field:

$$T(x, \tau) = \frac{Q}{2\pi\lambda} \sum_{k=1}^{n} \frac{1}{\tau - (k-1)\tau'} \exp\left\{-\frac{x^2}{4a[\tau - (k-1)\tau']}\right\},$$

$$\tau > (n-1)\tau', \tag{3}$$

and on the line $x = 0$

$$T(0, \tau) = \frac{Q}{2\pi\lambda} \sum_{k=1}^{n} \frac{1}{\tau - (k-1)\tau'}, \quad \tau > (n-1)\tau', \tag{4}$$

where $k = 1, 2, ..., n$ is the ordinal number of the pulse of thermal effect; n is the number of heat pulses.

If the instants of the formation of heat pulses are determined by conditions under which the preassigned relationships of temperatures are satisfied at a priori selected points of the surface $x = 0$ and $x = x'$, then the repetition rate of these pulses becomes dependent on the thermophysical characteristics of test materials [3].

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Actually, on termination of the nth thermal action at the time instant $\tau = n\tau'$ at a distance x' from the line of the disposition of the source, the excess temperature, according to Eq. (3), will take the value

$$T_{x',n\tau'} = \frac{Q}{2\pi\lambda\tau'} \sum_{k=1}^{n} \frac{1}{k} \exp\left(-\frac{x'^2}{4ak\tau'}\right), \tag{5}$$

and on the line of the disposition of the source

$$T_{0,n\tau'} = \frac{Q}{2\pi\lambda_1\tau'} \sum_{k=1}^{n} \frac{1}{k}. \tag{6}$$

The temperature ratio at the points $x = 0$ and $x = x'$ with allowance for Eqs. (6) and (5) at

$$\tau' = \frac{x'^2}{4a}. \tag{7}$$

is determined by the expression

$$\alpha_n = \frac{T_{0,n\tau'}}{T_{x',n\tau'}} = \frac{\sum_{k=1}^{n} \frac{1}{k}}{\sum_{k=1}^{n} \frac{1}{k} \exp\left(-\frac{1}{k}\right)}, \tag{8}$$

i.e., it can be predetermined.

Subjecting the material to short-term thermal effect from the source at the times when the temperatures are related in accordance with Eq. (8) and measuring the pulse repetition rate F, we can determine the thermal diffisivity of the material from Eq. (7):

$$a = \frac{Fx'^2}{4}. \tag{9}$$

For the determination of the thermal conductivity coefficient λ, it should be borne in mind that after the effect of the first heat pulse at the time when the temperatures are related according to Eq. (8):

$$\alpha_1 = 1/\exp(-1) \tag{10}$$

the temperature on the line $x = x'$ with account for Eq. (5) is

$$T_{x',\tau'} = \frac{Q}{2\pi\lambda\tau'} \exp(-1). \tag{11}$$

Thus, having fixed the value of the temperature on the line $x = x'$ at the time of attainment of relationship (10), we can determine the thermal conductivity coefficient from the formula

$$\lambda = \frac{FQ}{2\pi T_{x',\tau'}} \exp(-1). \tag{12}$$

To increase the accuracy of determining the thermal conductivity, it is worthwhile to record the values of temperature on the line $x = x'$ also on attainment of the relationships between $\alpha_2, \alpha_3, \ldots, \alpha_n$. Condition (7) being satisfied with allowance for Eq. (5), the ratios between the temperatures can be represented in the form

$$T_{x',\tau'} : T_{x',2\tau'} : T_{x',3\tau'} : \ldots : T_{x',n\tau'} = \exp(-1) : \left\{\frac{1}{2}\exp\left(-\frac{1}{2}\right) + \exp(-1)\right\} :$$

$$: \left\{\frac{1}{3}\exp\left(-\frac{1}{3}\right) + \frac{1}{2}\exp\left(-\frac{1}{2}\right) + \exp(-1)\right\} : \ldots : \sum_{k=1}^{n} \frac{1}{k}\exp\left(-\frac{1}{k}\right).$$

Then the value of temperature for calculating expression (12) is determined from the formula

$$\overline{T}_{x',\tau'} = \frac{\displaystyle\sum_{k=1}^{n} \left\{ \frac{T_{x',k\tau'} \exp(-1)}{\displaystyle\sum_{i=1}^{k} \frac{1}{i}\exp\left(-\frac{1}{i}\right)} \right\}}{n}, \tag{13}$$

where $i = 1, \ldots, k$; $k = 1, \ldots, n$.

The method described corresponds to the case of determining the parameters in repeated measurements and allows one to decrease the magnitude of the random component of the error of measurements by a factor of $\sqrt{n}$.

In determining the thermophysical characteristics, one should take into account systematic errors associated with the accuracy of assigning the amount of heat Q liberated by a heater and with the determination of the value of x'. The influence of these errors can be eliminated by performing preliminary standardization.

If a standard with known thermal diffusivity a_{st} and thermal conductivity λ_{st} is preliminarily tested as described above, then the following expressions are valid for its thermophysical coefficients:

$$a_{st} = \frac{F_{st} x'^2}{4}, \tag{14}$$

$$\lambda_{st} = \frac{F_{st} Q}{2\pi T_{stx',\tau'}} \exp(-1). \tag{15}$$

Having determined the quantities x'^2 and Q from Eqs. (14) and (15) and substituted them into Eqs. (9) and (12), respectively, we obtain the calculating expressions

$$a = a_{st} \frac{F_{st}}{F}, \tag{16}$$

$$\lambda = \lambda_{st} \frac{T_{stx',\tau'} F}{T_{x',\tau'} F_{st}}. \tag{17}$$

When materializing the above technique, the ideal model of the thermally semi-infinite material can be replaced by an actual model by adjusting the plane surface of the test material to the surface of another specimen (substrate) with high heat-insulating properties, resulting, however, in an additional procedural error. Therefore, evaluation of the value of this error is essential.

The temperature field in the boundary plane of two semi-infinite media with different thermophysical characteristics is described by the expression

$$T(x, \tau) = \frac{Q}{2\pi\tau(\lambda_1^2 - \lambda_2^2)} \left\{ \lambda_1 \exp\left(-\frac{x^2}{4a_1\tau}\right) - \lambda_2 \exp\left(-\frac{x^2}{4a_2\tau}\right) + \right.$$

$$+ \frac{x}{2\tau} \exp\left(-\frac{x^2}{4D\tau}\right) \left[\lambda_1 \left(\frac{1}{a_1} - \frac{1}{D}\right) \int_0^x \exp\left[-\left(\frac{1}{a_1} - \frac{1}{D}\right)\frac{\xi^2}{4\tau}\right] d\xi - \right.$$

$$-\lambda_2\left(\frac{1}{a_2}-\frac{1}{D}\right)\int_0^x \exp\left[-\left(\frac{1}{a_2}-\frac{1}{D}\right)\frac{\xi^2}{4\tau}\right]d\xi\Bigg]\Bigg\}, \quad \tau>0, \tag{18}$$

where $D = (\lambda_1^2 - \lambda_2^2)/((\lambda_1^2/a_1) - (\lambda_2^2/a_2))$ on condition that $\lambda_1^2/a_1 \neq \lambda_2^2/a_2$.

On the line of action of the heat source ($x = 0$), the magnitude of the excess temperature is described by the expression

$$T(0,\tau) = \frac{Q}{2\pi\tau(\lambda_1+\lambda_2)}, \quad \tau>0. \tag{19}$$

When the heat source forms a sequence of pulses in the time intervals τ', Eqs. (18) and (19) may correspondingly yield

$$T(x,\tau) = \frac{Q}{2\pi(\lambda_1^2-\lambda_2^2)}\sum_{k=1}^{n}\frac{1}{\tau-(k-1)\tau'}\left\{\lambda_1\exp\left(-\frac{x^2}{4a_1[\tau-(k-1)\tau']}\right)-\right.$$

$$-\lambda_2\exp\left(-\frac{x^2}{4a_2[\tau-(k-1)\tau']}\right)+\frac{x}{2[\tau-(k-1)\tau']}\times$$

$$\times\exp\left(-\frac{x^2}{4D[\tau-(k-1)\tau']}\right)\left[\lambda_1\left(\frac{1}{a_1}-\frac{1}{D}\right)\times\right.$$

$$\times\int_0^x\exp\left[-\left(\frac{1}{a_1}-\frac{1}{D}\right)\frac{\xi^2}{4[\tau-(k-1)\tau']}\right]d\xi-$$

$$\left.\left.-\lambda_2\left(\frac{1}{a_2}-\frac{1}{D}\right)\int_0^x\exp\left[-\left(\frac{1}{a_2}-\frac{1}{D}\right)\frac{\xi^2}{4[\tau-(k-1)\tau']}\right]d\xi\right]\right\}, \quad \tau>(n-1)\tau'. \tag{20}$$

$$T(0,\tau) = \frac{Q}{2\pi(\lambda_1+\lambda_2)}\sum_{k=1}^{n}\frac{1}{\tau-(k-1)\tau'}, \quad \tau>(n-1)\tau'. \tag{21}$$

In performing the numerical simulation of heat-transfer processes, we calculated excess temperatures at control points from formulas (20)-(21) and (3)-(4) with allowance for the satisfaction of preassigned relations (8) at $Q = 20$ J/m and $x' = 2.5\cdot10^{-3}$ m and fixed the time of the test and the control values of temperatures and, proceeding from the number of heat pulses, determined their repetition rate. On the basis of Eqs. (9), (12), and (13), we calculated the thermophysical coefficients and the values of errors:

$$\delta a = \frac{a_R - a_I}{a_I}\cdot 100\%, \tag{22}$$

$$\delta\lambda = \frac{\lambda_R - \lambda_I}{\lambda_I}\cdot 100\%, \tag{23}$$

where the subscripts R and I denote the parameters calculated for the actual and ideal models of the thermally semi-infinite medium.

The values of the thermophysical coefficients for the substrate material were selected to correspond to a specific specimen tested (rippor) $a_r = 3.13\cdot10^{-7}$ m^2/sec and $\lambda_r = 0.026$ W/(m·K).

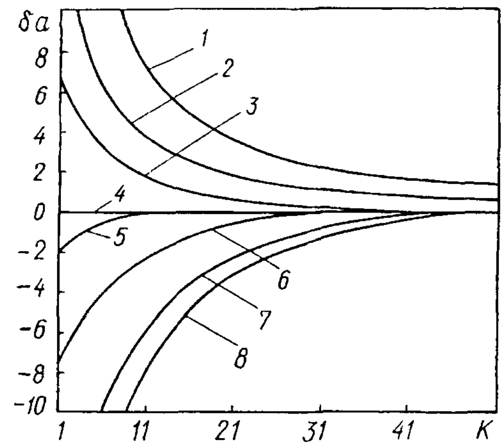

Fig. 1. Graphs of the dependence of the error in determining the thermal diffusivity on the ratio between the thermal conductivities of the test material and the substrate material: 1) at $a = 10^{-7}$; 2) $2 \cdot 10^{-7}$; 3) $2.5 \cdot 10^{-7}$; 4) $3.13 \cdot 10^{-7}$; 5) $3.5 \cdot 10^{-7}$; 6) $5 \cdot 10^{-7}$; 7) $7.5 \cdot 10^{-7}$; 8) 10^{-6} m^2/sec. δa, %.

Figure 1 presents the graphs of the relative error in the thermal diffusivity of the test material δa as a function of the ratio between the thermal conductivities of the test material and of the specimen, $K = \lambda/\lambda_r$ ($K > 1$). When $a = a_r$, the graph is a straight line coinciding with the abscissa axis. Actually, when $a_1 = a_2$, Eq. (20) is rearranged to the form

$$T(x, \tau) = \frac{Q}{2\pi(\lambda_1 + \lambda_2)} \sum_{k=1}^{n} \frac{1}{\tau - (k-1)\tau'} \exp\left\{-\frac{x^2}{4a_1[\tau - (k-1)\tau']}\right\}, \quad \tau > (n-1)\tau'. \tag{24}$$

On termination of the nth thermal effect at the time instant $\tau = n\tau'$ at a distance of x' from the source, the excess temperature takes on (according to Eq. (24)) the value

$$T_{x',n\tau'} = \frac{Q}{2\pi(\lambda_1 + \lambda_2)\tau'} \sum_{k=1}^{n} \frac{1}{k} \exp\left(-\frac{x'^2}{4a_1 k\tau'}\right), \tag{25}$$

while on the line of the disposition of the heat source, the excess temperature, according to Eq. (21), is

$$T_{0,n\tau'} = \frac{Q}{2\pi(\lambda_1 + \lambda_2)\tau'} \sum_{k=1}^{n} \frac{1}{k}. \tag{26}$$

When $\tau' = x'^2/4a_1$, the ratio between temperatures (26) and (25), just as between temperatures (6) and (5), is described by formula (8).

Thus, for two contacting semi-infinite materials with different thermophysical characteristics, the preassigned relationships between the temperatures are satisfied in the same time intervals as for the thermally semi-infinite material, and consequently the repetition rate of heat pulses is the same. According to Eq. (9), the calculated values of the thermal diffusivity coincide in both cases.

Calculations showed that for $a < a_r$ the assigned relationships for the temperatures (21) and (20) are satisfied in the intervals $\tau' < x'^2/4a$, i.e., the repetition rate of heat pulses for the actual model is higher than for the ideal one. From this, according to Eqs. (9) and (22), it follows that $\delta a > 0$.

When $a > a_r$, the assigned relationships for the temperatures (21) and (20) are satisfied in the intervals $\tau' > x'^2/4a$; therefore, $\delta a < 0$.

For test materials with $\lambda \gg \lambda_r$ the actual model is close to the ideal one. Indeed, when $\lambda_1 \gg \lambda_2$, we can take the quantity λ_2 in Eqs. (20) and (21) to be equal to zero, and the expressions are transformed into Eqs. (3) and (4).

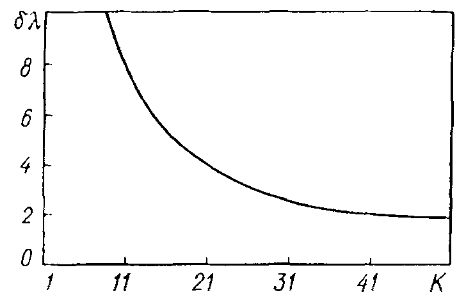

Fig. 2. Graph of the dependence of the error in determining the thermal conductivity on the ratio between the thermal conductivities of the test material and the substrate material. $\delta\lambda$, %.

TABLE 1. Results of Experimental Determination of Errors in Measuring the Thermal Diffusivity and Thermal Conductivity of Materials

Parameters		Material 1	2	3	4
Thermal diffusivity $a \cdot 10^7$, m^2/sec		1.10	3.41	1.78	3.80
Thermal conductivity λ, W/(m·K)		0.208	0.252	0.418	1.005
Ratio $K=\lambda/\lambda_r$		8	9.69	16.1	38.7
Difference $\lvert a - a_r \rvert \cdot 10^7$, m^2/sec		2.03	0.28	1.35	0.67
Systematic component of relative error at $n = 1$	$\delta a_{sys.}$, %	10.7	1.2	4.1	1.0
Systematic component of relative error at $n = 1$	$\delta\lambda_{sys.}$, %	10.0	8.2	5.3	3.0
Random component of relative error at $n = 1$	$\delta a_{ran.}$, %	8.5	8.6	8.7	8.6
Random component of relative error at $n = 1$	$\delta\lambda_{ran.}$, %	9.0	8.8	8.9	8.8
Relative error at $n = 1$	δa, %	19.2	9.8	12.8	9.6
Relative error at $n = 1$	$\delta\lambda$, %	19.0	17.0	15.2	11.8
Systematic component of relative error at $n = 10$	$\delta a_{sys.}$, %	10.9	1.3	4.2	1.1
Systematic component of relative error at $n = 10$	$\delta\lambda_{sys.}$, %	10.5	8.2	6.5	3.1
Random component of relative error at $n = 10$	$\delta a_{ran.}$, %	4.2	4.1	4.3	4.1
Random component of relative error at $n = 10$	$\delta\lambda_{ran.}$, %	4.4	4.2	4.4	4.2
Relative error at $n = 10$	δa, %	15.1	5.4	8.5	5.2
Relative error at $n = 10$	$\delta\lambda$, %	14.9	12.4	11.0	7.3
Ratio $\delta a_{ran.}\rvert_{n=1} : \delta a_{ran.}\rvert_{n=10}$		2.02	2.10	2.01	2.10
Ratio $\delta\lambda_{ran.}\rvert_{n=1} : \delta\lambda_{ran.}\rvert_{n=10}$		2.05	2.09	2.02	2.09

Note: 1) Polymethylmethacrylate, 2) Getinaks, 3) fiberylass-base laminate, 4) aluminum-graphite composition based epoxidy resin.

Figure 2 gives the graph for the relative error of the thermal conductivity of the test material $\delta\lambda$ versus the ratio of thermal conductivities K. The magnitude of the thermal diffusivity of the test material has no effect on the behavior of the dependence.

The calculations showed that for the materials with the same values of λ, but with different values of a, the ratio $F/T_{x',n\tau'}$ is a constant quantity. According to Eqs. (12) and (23), the quantity $\delta\lambda$ for such materials takes on the same and, moreover, positive value, since

$$(F/T_{x',n\tau'})_R > (F/T_{x',n\tau'})_I .$$

Table 1 presents the results for the experimental determination of relative errors in measuring the thermophysical characteristics of different materials with the number of heat pulses $n = 1$ and $n = 10$. The measurements were made by an automatic device, developed and constructed on the basis of a Vektor personal computer [4], according to the procedure given in [5]. A sample of rippor with $a_r = 3.13 \cdot 10^{-7}$ m^2/sec and $\lambda_r = 0.026$ W/(m·K) was used as a substrate.

The use of the frequency-pulse method ($n = 10$) entailed a decrease in the mean-square deviation of measurements by more than a factor of two, which caused a reduction in the random component of measurement errors as compared with those occurring in the case of single thermal effect.

Comparison of the dependence of the values of the systematic component in the error of measurements on the values of the ratio of thermal conductivities and differences of thermal diffusivities of the test materials and substrate with the results of numerical simulation confirms the reliability of the investigations. The systematic component of the error is basically of a procedural nature, and its magnitude is independent of the number of pulses exerting a thermal effect. Therefore, to eliminate it, one can interpolate the curves for the dependences of errors, for example, by the Lagrange polynomial. Depending on the result of measurements, it is possible to automatically use a corresponding function, stored in the computer memory, as well as to compute and introduce a correction, which, together with the simultaneous use of preliminary standardization, will allow one to perform measurements with an error not exceeding 5%.

NOTATION

x, y, coordinates, m; τ, time, sec; T, temperature, °C, K; λ, thermal conductivity coefficient, W/(m·K); a, thermal diffusivity coefficient, m^2/sec; Q, amount of heat per unit length, J/m.

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